

**WILLETTON SENIOR HIGH SCHOOL**  
**Semester One Examination, 2022**  
**Question/Answer booklet**

**MATHEMATICS**  
**APPLICATIONS**  
**UNIT 1**

**Section Two:**  
**Calculator-assumed**

**SOLUTIONS**

Your name \_\_\_\_\_

Teacher's name \_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: ten minutes  
Working time: one hundred minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer booklet  
Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
<b>Total</b>					100

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (98 Marks)

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

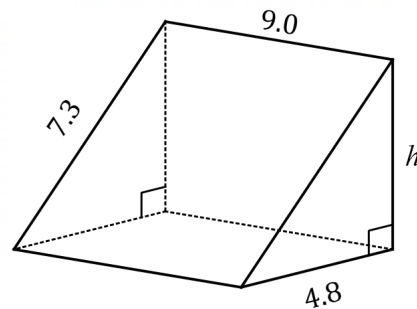
Working time: 100 minutes.

**Question 8**

(6 marks)

The diagram, not to scale, shows a right triangular prism with a length of 9.0 cm.

The lengths of some of the edges are shown on the diagram in centimetres.



(a) Determine the height  $h$  of the right triangle.

(2 marks)

<b>Solution</b>
$h^2 = 7.3^2 - 4.8^2$ $= 30.25$ $h = 5.5 \text{ cm}$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ shows correct use of Pythagoras' Theorem</li> <li>✓ correct height</li> </ul>

(b) Determine the total surface area of the prism.

(4 marks)

<b>Solution</b>
<p>Area of triangular end:</p> $A_1 = \frac{1}{2} \times 4.8 \times 5.5$ $= 13.2 \text{ cm}^2$ <p>Area of rectangles:</p> $A_2 = 9.0 \times (4.8 + 5.5 + 7.3)$ $= 9 \times 17.6$ $= 158.4 \text{ cm}^2$ <p>Total surface area:</p> $A = 2 \times 13.2 + 158.4$ $= 184.8 \text{ cm}^2$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ calculates area of one triangle</li> <li>✓ calculates area of at least one rectangle</li> <li>✓ indicates sum of all five faces</li> <li>✓ correct total surface area</li> </ul>

Question 9

(9 marks)

Ash lives at home with her parents. Her take-home income and all the expenses used in her personal budget from the previous year are shown below, together with how often the amounts are incurred and updates that Ash needs to make for the current year.

Budget item	Previous amount (\$)	Frequency	Update
Take-home income	2100.00	Fortnightly	Increase by 3.5%
Gym membership	22.00	Weekly	Increase by \$3.50
Household contribution	140.00	Weekly	Increase by \$30
Mobile phone	45.25	Monthly	No change
Car running costs	150.00	Weekly	Decrease by 10%
Car loan repayment	506.75	Monthly	No change
Entertainment & eating out	150.00	Weekly	Increase by 5%
Other	450.00	Monthly	Decrease by 5%

- (a) Determine the amount of her fortnightly take-home income for the current year, after the change indicated in the update column. (2 marks)

Solution
$I = 1.035 \times 2100$ $= \$2173.50$
Specific behaviours
✓ indicates correct method ✓ correct amount

- (b) Use the information in the update column to complete the missing entries in the table below for Ash's budget for the current year. (3 marks)

Budget item	Amount this year (\$)	Frequency
Take-home income	<b>2173.50</b>	Fortnightly
Gym membership	<b>25.50</b>	Weekly
Household contribution	<b>170.00</b>	Weekly
Mobile phone	45.25	Monthly
Car running costs	<b>135.00</b>	Weekly
Car loan repayment	506.75	Monthly
Entertainment & eating out	<b>157.50</b>	Weekly
Other	427.50	Monthly

Solution
$\$2173.50,$ $22.00 + 3.50 = \$25.50,$ $140.00 + 30.00 = \$170.00$  $150.00 \times 0.90 = \$135.00,$ $150 \times 1.05 = \$157.50$
Specific behaviours
✓ first three entries (income and flat increases) correct ✓ fourth entry (% decrease) correct ✓ fifth entry (% increase) correct

After subtracting her expenses from her take-home income, Ash calculated (assuming that there are exactly 52 weeks in a year and 26 fortnights in a year) that at the end of last year she was left with a surplus of \$18 552 to use for a holiday or deposit in a savings account.

- (c) Ash hopes that her budget updates for the current year will lead to an increase in her annual surplus. Determine, with justification, whether this is the case. (4 marks)

<b>Solution</b>	
Income:	$I = 2173.50 \times 26 = \$56\,511$
Weekly costs:	$C_W = (25.50 + 170.00 + 135.00 + 157.50) \times 52$ $= 488 \times 52$ $= \$25\,376$
Monthly costs:	$C_M = (45.25 + 506.75 + 427.50) \times 12$ $= 979.50 \times 12$ $= \$11\,754$
Annual surplus:	$S = 56\,511 - (25\,376 + 11\,754)$ $= 56\,511 - 37\,130$ $= \$19\,381$
Hence her annual surplus has slightly increased from \$18 552 to \$19 381 - which is as she had hoped.	
<b>Specific behaviours</b>	
<ul style="list-style-type: none"> <li>✓ correctly adjusts fortnightly income to annual</li> <li>✓ correctly adjusts weekly costs to annual</li> <li>✓ correctly adjusts monthly costs to annual</li> <li>✓ calculates surplus and notes it has increased</li> </ul>	

Question 10

(8 marks)

At the start of this year, an art supplies wholesaler in Australia needed to buy 4800 flat brushes and collected the following price information from overseas manufacturers:

Country of manufacturer	Poland	Singapore	Mexico
Number of flat brushes per pack	320	400	300
Cost per pack (in local currency, includes shipping)	1870 zloty	\$850	9300 pesos

At the time, the relevant exchange rates for one Australian dollar were 2.855 Polish zloty, 0.948 Singapore dollars and 14.42 Mexican pesos.

- (a) Determine the cost per brush from the Polish manufacturer in Australian dollars. (2 marks)

<b>Solution</b>
Price in Australian dollars: $1870 \div 2.855 = \$654.99$
Unit price: $654.99 \div 320 = \$2.05$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ converts foreign price to Australian dollars</li> <li>✓ calculates unit cost</li> </ul>

- (b) Determine the total cost in Australian dollars of buying 4800 brushes from the Mexican manufacturer. (3 marks)

<b>Solution</b>
Will need $4800 \div 300 = 16$ packs.
Cost in Mexican pesos: $16 \times 9300 = 148\,800$ .
Cost in Australian dollars: $148\,800 \div 14.42 = \$10\,319.00$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ indicates number of packs required</li> <li>✓ indicates total cost in local currency</li> <li>✓ calculates total cost in Australian dollars</li> </ul>

- (c) Determine, with justification, which of the three countries the wholesaler should use in order to minimise the unit cost per brush. (3 marks)

<b>Solution</b>
Unit price for Mexican manufacturer: $10\,319 \div 4800 = \$2.15$
Singapore pack price in Australian dollars: $850 \div 0.948 = \$896.62$ Singapore unit price: $896.62 \div 400 = \$2.24$
Hence unit cost per brush from Poland, Singapore, Mexico is \$2.05, \$2.24, \$2.15 and so for minimum cost the wholesaler should choose the Polish manufacturer.
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ indicates unit price for Mexican manufacturer</li> <li>✓ indicates unit price for Singapore manufacturer</li> <li>✓ correctly identifies country for minimum unit cost</li> </ul>

Question 11

(8 marks)

Information about the shares of three companies listed on the Australian Securities Exchange (ASX) towards the end of last year is shown in the table below.

ASX Share Code	Market value of share (\$)	Earnings per share (\$)	Dividend per share (\$)	Percentage dividend
HVN	5.07	0.674	0.35	6.90%
MPL	3.42	0.160	<b>A</b>	3.80%
DMP	120.83	2.119	1.74	<b>B</b>

- (a) Determine the value of **A** and the value of **B** in the table. (2 marks)

<b>Solution</b>
$A = 3.42 \times 3.80\% = 0.13$
$B = 1.74 \div 120.83 = 1.44\%$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ value of <b>A</b></li> <li>✓ value of <b>B</b></li> </ul>

- (b) Calculate the price-to-earnings ratio for each share and hence rank the stocks in order from highest to lowest price relative to earnings. (3 marks)

<b>Solution</b>
P/E ratios: HVN: $5.07 \div 0.674 = 7.52$ MPL: $3.42 \div 0.160 = 21.38$ DMP: $120.83 \div 2.119 = 57.02$
Ranking from highest to lowest is DMP, MPL, HVN.
<i>(Accuracy: Accept ratios that round to 8, 21 and 57)</i>
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ correctly calculates at least one P/E ratio</li> <li>✓ all ratios correct</li> <li>✓ correctly ranks in order from highest to lowest</li> </ul>

- (c) Calculate, to the nearest dollar, the total dividend paid to an investor who owned 684 shares in HVN, 1240 shares in MPL and 215 shares in DMP. (3 marks)

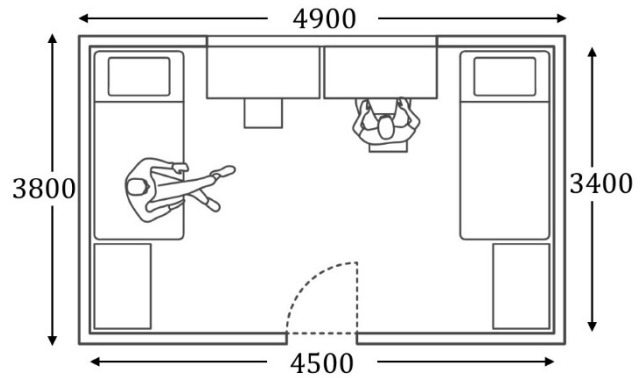
<b>Solution</b>
Individual stock dividends: HVN: $684 \times 0.35 = \$239.40$ or $5.07 \times 684 \times 0.069 = \$239.28$ MPL: $1240 \times 0.13 = \$161.20$ or $3.42 \times 1240 \times 0.038 = \$161.15$ DMP: $215 \times 1.74 = \$374.10$ or $120.83 \times 215 \times 0.0144 = \$374.09$
Totals: \$774.70, \$774.52 or \$774.65. Hence total dividend is \$775 to nearest dollar.
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ correctly calculates dividend for at least one stock</li> <li>✓ all stock dividends correct (using either method)</li> <li>✓ correct total dividend</li> </ul>

**Question 12**

(8 marks)

A sketch plan of a twin bedroom is shown, not drawn to scale.

The internal and external dimensions of the room are shown on the sketch in millimetres.



- (a) State the internal length and width of the room in metres. (2 marks)

Solution
$l = 4.5 \text{ m}, \quad w = 3.4 \text{ m}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ uses correct dimensions (<math>l</math> and <math>w</math> interchangeable)</li> <li>✓ writes dimensions in metres</li> </ul>

The floor of the interior of the room is to be carpeted using a 10 m long roll of carpet that is 1.8 m wide. No more than four cuts can be made to the carpet.

- (b) Sketch a diagram to show how the carpet could be laid on the floor, showing the dimensions of each piece of carpet cut from the roll. (3 marks)

Solution
Dimensions of A: $4.5 \times 1.8 \text{ m}$ , of B: $4.5 \times 1.6 \text{ m}$ .
Specific behaviours
<ul style="list-style-type: none"> <li>✓ suitably subdivided rectangle</li> <li>✓ one suitably dimensioned piece of carpet</li> <li>✓ all pieces of carpet suitably dimensioned</li> </ul>

- (c) After the floor has been carpeted, some of the roll is left over. Determine what percentage of the area of the roll is left over. (3 marks)

Solution
<p>Area of roll: <math>A_R = 10 \times 1.8 = 18.0 \text{ m}^2</math>                  Area of floor: <math>A_F = 4.5 \times 3.4 = 15.3 \text{ m}^2</math>                  Area left over: <math>A = 18.0 - 15.3 = 2.7 \text{ m}^2</math></p> <p>Percentage left over: <math>2.7 \div 18 = 15\%</math></p>
Specific behaviours
<ul style="list-style-type: none"> <li>✓ areas of floor and roll</li> <li>✓ shows correct calculation for percentage</li> <li>✓ evaluates percentage (correct to nearest whole number)</li> </ul>



**Question 13**

(8 marks)

The image shows some recently harvested grain stored in a conical pile on level ground.



The top of the pile is 12 m above the ground and the radius of the base is 35 m.

- (a) Determine the volume of grain contained in the pile. (2 marks)

Solution
$V = \frac{1}{3} \times \pi \times 35^2 \times 12$ $= 15\,394 \text{ m}^3$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ shows correct use of volume formula</li> <li>✓ calculates volume (correct to at least 3 sf)</li> </ul>

- (b) Trucks that can each carry a load of 28 tonnes will be used to transport the grain to the local port. If one cubic metre of grain weighs 630 kg, determine the number of truck loads of grain required to move the entire pile of grain to the port. Note that there are 1000 kg in one tonne. (2 marks)

Solution
$W = 15\,394 \times 630 \div 1000$ $= 9698 \text{ tonnes}$ $9698 \div 28 = 346.4$ <p>Hence will need 347 truck loads.</p>
Specific behaviours
<ul style="list-style-type: none"> <li>✓ calculates weight of grain in tonnes</li> <li>✓ calculates number of trucks</li> </ul>

- (c) Determine the slant height of the conical pile. (2 marks)

Solution
$s = \sqrt{12^2 + 35^2}$ $= \sqrt{1369} = 37 \text{ m}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ shows correct use of Pythagoras' Theorem</li> <li>✓ calculates correct slant height</li> </ul>

- (d) When rain is forecast, the sloping surface of the grain can be covered with tarpaulins. Calculate the area of grain that needs to be covered. (2 marks)

Solution
$A = \pi \times 35 \times 37$ $= 4068 \text{ m}^2$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ uses correct part of surface area formula</li> <li>✓ calculates area</li> </ul>

Question 14

(7 marks)

The table below and matrix **T** show the number of trips (in thousands) made in 2005 by visitors to three Australian cities categorised by reason (holiday or visiting friends / relatives).

(000's)	Holiday	VFR
Sydney	1952	2718
Adelaide	576	790
Perth	854	1107

$$\mathbf{T} = \begin{bmatrix} 1952 & 2718 \\ 576 & 790 \\ 854 & 1107 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} 1.035 & 0 & 0 \\ 0 & 1.02 & 0 \\ 0 & 0 & 1.03 \end{bmatrix}$$

Visitor numbers in both categories to Sydney, Adelaide and Perth increased by 3.5%, 2% and 3% per year respectively over the following years and this information is shown in matrix **R**.

- (a) Determine matrix  $\mathbf{R} \times \mathbf{T}$ , rounding entries to the nearest whole number. (2 marks)

<b>Solution</b>	
$\mathbf{R} \times \mathbf{T} =$	$\begin{bmatrix} 2020 & 2813 \\ 588 & 806 \\ 880 & 1140 \end{bmatrix}$
<b>Specific behaviours</b>	
<ul style="list-style-type: none"> <li>✓ at least one row is correct</li> <li>✓ correct matrix, rounded</li> </ul>	

- (b) How many more trips for a holiday were made to Perth in 2006 (the following year) compared to the same type of trip to Adelaide in 2006? (2 marks)

<b>Solution</b>	
$880 - 588 = 292$	
There were 292 000 more trips.	
<b>Specific behaviours</b>	
<ul style="list-style-type: none"> <li>✓ identifies both required entries</li> <li>✓ states difference of trips adjusting for thousands</li> </ul>	

- (c) The matrix  $\mathbf{R}^n \times \mathbf{T}$  will show the number of trips (in thousands) by category in the  $n^{th}$  year after 2005. Determine, with justification, the year in which the number of VFR trips to Adelaide first exceeded 900 000. (3 marks)

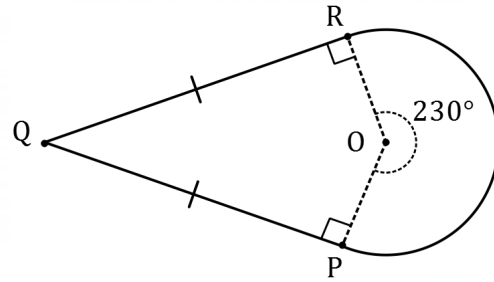
<b>Solution</b>	
$\mathbf{R}^7 \times \mathbf{T} =$	$\begin{bmatrix} 2483 & 3458 \\ 662 & \mathbf{907} \\ 1050 & 1361 \end{bmatrix}$
or solve $790 \times 1.02^n = 900$ .	
$n = 6.6$	
Hence after 7 years, in 2012.	
<b>Specific behaviours</b>	
<ul style="list-style-type: none"> <li>✓ indicates value of <math>n</math></li> <li>✓ states correct year</li> <li>✓ justification (shows matrix, element or other method)</li> </ul>	

**Question 15**

(8 marks)

The diagram (not to scale) shows a fabric panel used in the construction of a kite.

The panel consists of a  $230^\circ$  sector of a circle with centre  $O$  and radius 50 cm, and quadrilateral  $OPQR$  in which  $QR = QP = 108$  cm.



(a) Calculate the area of the sector.

(2 marks)

Solution
$A = \frac{230}{360} \times \pi \times 50^2 = 5017.82 \text{ cm}^2$ <p>(Accepted 5018 cm<sup>2</sup>)</p>
Specific behaviours
<ul style="list-style-type: none"> <li>✓ uses appropriate method</li> <li>✓ calculates sector area</li> </ul>

(b) Hence determine the total area of the fabric panel.

(3 marks)

Solution
Quadrilateral can be split into two right triangles. Area of a triangle: $A = \frac{1}{2} \times 50 \times 108 = 2700 \text{ cm}^2$ Hence total area is sector plus two triangles: $A = 5017.82 + 2 \times 2700 = 10\,417.82 \text{ cm}^2$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ splits quadrilateral into two right-triangles</li> <li>✓ area of one right-triangle</li> <li>✓ calculates total area</li> </ul>

(c) The panel is to be edged with tape. Calculate the length of tape required.

(3 marks)

Solution
Perimeter of sector, $P$ : $P = \frac{230}{360} \times 2 \times \pi \times 50 = 200.71 \text{ cm}$
Total length of tape, $L$ : $L = 200.71 + 2 \times 108$ $= 416.71 \text{ cm}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ indicates correct method for sector perimeter</li> <li>✓ calculates sector perimeter</li> <li>✓ calculates total length of tape</li> </ul>

**Question 16**

**(8 marks)**

A single person aged 18 to 21, living away from their parents and looking for work can qualify for a youth allowance from the government of \$512.50 per fortnight, provided their fortnightly income is no more than \$150.

In any fortnight that a person earns more than \$150, the youth allowance is reduced by 50 cents for each dollar of income they have between \$150 and \$250. If their income is over \$250, the payment will further reduce by 60 cents for each dollar of income over \$250.

Emily qualifies for the allowance. She has a casual job that pays \$19.50 per hour, working for three-and-a-half hours every Wednesday and Friday at a bakery and she also receives 100 Euros each week for work she does for an online business in Europe.

- (a) Calculate Emily's fortnightly earnings from her bakery job. (2 marks)

<b>Solution</b>
Hours worked per fortnight: $3.5 \times 2 \times 2 = 14$ Fortnightly earnings: $14 \times 19.50 = \$273$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ indicates correct number of hours worked per fortnight</li> <li>✓ calculates fortnightly earnings</li> </ul>

- (b) The government use an exchange rate of one Australian dollar to 0.6734 Euros for foreign earnings. Calculate the fortnightly amount that Emily earns from the European business. (2 marks)

<b>Solution</b>
Australian dollars: $100 \div 0.6734 = 148.50$ Fortnightly earnings: $2 \times 148.50 = \$297$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ correctly uses exchange rate</li> <li>✓ calculates fortnightly earnings</li> </ul>

- (c) Determine Emily's total fortnightly income from her bakery job, European business and government allowance. (4 marks)

<b>Solution</b>
Total fortnightly earnings: $273.00 + 297.00 = \$570.00$  First reduction: $(250 - 150) \times 0.50 = \$50.00$ Next reduction: $(570 - 250) \times 0.60 = \$192.00$  Government allowance: $512.50 - 50.00 - 192.00 = \$270.50$  Total fortnightly income: $570.00 + 270.50 = \$840.50$ .
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ indicates total fortnightly earnings</li> <li>✓ indicates both reductions</li> <li>✓ calculates government allowance</li> <li>✓ calculates total income</li> </ul>

Question 17

(10 marks)

- (a) Kelly takes a short-term loan of \$2400 at 9.6% per annum simple interest for 50 days. Determine the sum of the principal and interest that Kelly must repay at the end of the loan period. (3 marks)

<b>Solution</b>
$I = \frac{2400 \times 9.6 \times 50}{100 \times 365} = 31.56$
Repay $2400 + 31.56 = \$2431.56$ .
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ correct expression to calculate simple interest</li> <li>✓ calculates simple interest</li> <li>✓ calculates sum to repay</li> </ul>

- (b) An amount of \$15 000 is invested at 4.5% per annum compounded annually for 3 years. Determine the total interest earned on this investment over the 3 years. (3 marks)

<b>Solution</b>
$A = 15\,000(1.045)^3 = 17\,117.49$
Interest earned is $17\,117.49 - 15\,000 = \$2117.49$ .
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ correct expression to calculate future value</li> <li>✓ calculates future value</li> <li>✓ calculates interest earned</li> </ul>

- (c) At the start of last year, Syed was employed as a manager with a salary of \$86 000. During the year, Syed was awarded a 6.5% pay rise followed by another rise of 7.7% at the start of this year.

- (i) Determine Syed's salary at the start of this year. (2 marks)

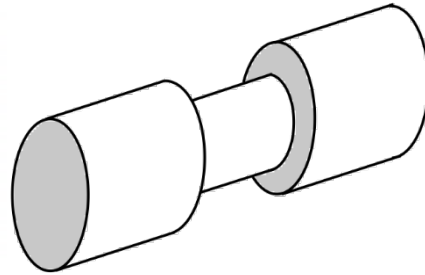
<b>Solution</b>
$S = 86\,000 \times 1.065 \times 1.077 = \$98\,642.43$
or
$86\,000 \times 1.065 = \$91\,590$
$91\,590 \times 1.077 = \$98\,642.43$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ indicates correct method</li> <li>✓ calculates new salary</li> </ul>

- (ii) Determine the percentage increase in Syed's salary from the start of last year to the start of this year. (2 marks)

<b>Solution</b>
$\frac{98\,642.43 - 86\,000}{86\,000} = \frac{12\,642.43}{86\,000} = 14.7\% \text{ increase.}$
or
$1.065 \times 1.077 = 1.147005 \rightarrow 14.7\% \text{ increase.}$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ indicates correct method</li> <li>✓ correct percentage increase</li> </ul>

**Question 18****(9 marks)**

A woodturner takes a solid cylindrical wooden pole of length 66 cm and radius 8 cm and uses a machine to reduce the radius of the middle third of the pole by 30%, as shown in the diagram (not to scale).



- (a) Explaining your method, calculate the volume of wood removed by the machine.

**(4 marks)**

<b>Solution</b>
Volume removed will be difference in volume of middle section of pole as a cylinder with height of $66 \div 3 = 22$ cm and initial radius of 8 cm and final radius of $8 \times 0.7 = 5.6$ cm. $V_1 = \pi \times 8^2 \times 22 = 4423 \text{ cm}^3$ $V_2 = \pi \times 5.6^2 \times 22 = 2167 \text{ cm}^3$ Hence volume removed is $4423 - 2167 = 2256 \text{ cm}^3$ .
<b>Specific behaviours</b>
<ul style="list-style-type: none"><li>✓ explanation of method or describes each step</li><li>✓ correctly calculates volume of original cylinder <math>V_1</math></li><li>✓ correctly calculates volume of final cylinder <math>V_2</math></li><li>✓ correctly calculates volume removed</li></ul>

- (b) Explaining your method, determine the total surface area of the remaining object. (5 marks)

<b>Solution</b>
<p>TSA of one large end cylinder:</p> $\begin{aligned} A_1 &= 2\pi \times 8^2 + 2\pi \times 8 \times 22 \\ &= 402.1 + 1105.84 \\ &= 1508 \text{ cm}^2 \end{aligned}$
<p>Area of curved surface of small middle cylinder:</p> $A_2 = 2\pi \times 5.6 \times 22 = 774 \text{ cm}^2$
<p>Area of one circular end of small middle cylinder:</p> $A_3 = \pi \times 5.6^2 = 98.5 \text{ cm}^2$
<p>TSA of remaining object:</p> $\begin{aligned} A &= 2A_1 + A_2 - 2A_3 \\ &= 3016 + 774 - 197 \\ &= 3593 \text{ cm}^2 \end{aligned}$
OR
<p>TSA of two large cylinders:</p> $\begin{aligned} A_1 &= 2\pi \times 8^2 + (2\pi \times 8 \times 22) \times 2 \\ &= 402.1 + 2211.68 \\ &= 2613.78 \text{ cm}^2 \end{aligned}$
<p>Area of curved surface of small middle cylinder:</p> $A_2 = 2\pi \times 5.6 \times 22 = 774 \text{ cm}^2$
<p>Area of two 'donuts' joining small and large cylinder:</p> $A_3 = (\pi \times 8^2 - \pi \times 5.6^2) \times 2 = 205.08 \text{ cm}^2$
<p>TSA of remaining object:</p> $\begin{aligned} A &= A_1 + A_2 + A_3 \\ &= 2614 + 774 - 205 \\ &= 3593 \text{ cm}^2 \end{aligned}$
<p>Many possible methods.</p>
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ explanation of method or describes each step</li> <li>✓ correctly calculates TSA of larger cylinder <math>A_1</math></li> <li>✓ correctly calculates curved surface of smaller cylinder <math>A_2</math></li> <li>✓ correctly calculates area of smaller cylinder ends <math>A_3</math></li> <li>✓ correct TSA of object</li> </ul>

## Question 19

(9 marks)

When a person needs to save an amount of  $A$  dollars by making  $N$  monthly deposits into an account with an interest at a rate of  $R\%$  p.a. compounded monthly, then the deposit that they must make each month,  $D$  dollars, is given by the formula

$$D = \frac{Ai}{k}, \text{ where } i = \frac{R}{1200} \text{ and } k = (1 + i)^N - 1.$$

- (a) Lydia will make monthly deposits into a savings account with an interest rate of 6.96% p.a. compounded monthly and wants to save \$18 000 to buy a used car in two years' time.
- (i) How many monthly deposits will she make in two years? (1 mark)

Solution
$N = 12 \times 2 = 24$ deposits
Specific behaviours
✓ correct number

- (ii) Determine the amount that she must deposit each month. *Hint: Do not round the value of  $k$  in your calculations.* (4 marks)

Solution
$i = \frac{6.96}{1200} = 0.0058, \quad k = (1 + 0.0058)^{24} - 1 = 0.14889$
$D = \frac{18\,000 \times 0.0058}{0.14889} = 701.18$
Hence the monthly deposit is \$701.18.
Specific behaviours
<ul style="list-style-type: none"> <li>✓ calculates value of <math>i</math></li> <li>✓ calculates value of <math>k</math></li> <li>✓ indicates expression for <math>D</math></li> <li>✓ correctly evaluates <math>D</math></li> </ul>



- (b) Josh has no savings but has just started a job and can afford to deposit \$500 per month in a savings account with an interest rate of 7.8% p.a. compounded monthly. After working for six years, he hopes to have saved \$45 000 so that he can take a year off work and travel around the world. Comment on how feasible his savings plan is to achieve this aim. (4 marks)

<b>Solution</b>
<p>Number of deposits, <math>N = 12 \times 6 = 72</math>.</p> $i = \frac{7.8}{1200} = 0.0065, \quad k = (1 + 0.0065)^{72} - 1 = 0.59438$ $D = \frac{45\,000 \times 0.0065}{0.59438} = 492.11$ <p>Depositing \$492.11 per month will ensure Josh saves \$45 000 and since he can afford \$500, then he will meet this aim.</p> <p>OR</p> <p>Number of deposits, <math>N = 12 \times 6 = 72</math>.</p> $i = \frac{7.8}{1200} = 0.0065, \quad k = (1 + 0.0065)^{72} - 1 = 0.59438$ $500 = \frac{A \times 0.0065}{0.59438}$ $A = \$45\,721.54$ <p>Depositing \$500 per month will allow Josh to save \$45 721.54 which exceeds the \$45 000 he requires, therefore he will meet his aim.</p>
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ indicates number of deposits <math>N</math> and value of <math>i</math></li> <li>✓ calculates value of <math>k</math></li> <li>✓ evaluates <math>D</math></li> <li>✓ relevant comment on feasibility of plan</li> </ul>

Supplementary page

Question number: \_\_\_\_\_

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